



# Bisimilarity Enforcing Supervisory Control for Deterministic Specifications <sup>☆</sup>

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## Abstract

This paper investigates the supervisory control of nondeterministic discrete event systems to enforce bisimilarity with respect to deterministic specifications. A notion of synchronous simulation-based controllability is introduced as a necessary and sufficient condition for the existence of a bisimilarity enforcing supervisor, and a polynomial algorithm is developed to verify such a condition. When the existence condition holds, a supervisor achieving bisimulation equivalence is constructed. Furthermore, when the existence condition does not hold, two different methods are provided for synthesizing maximal permissive sub-specifications.

*Keywords:* Supervisory control, bisimulation, discrete event systems

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## 1. INTRODUCTION

The notion of bisimulation introduced by Milner (1989) has been successfully used as a behavior equivalence in model checking (Clarke, 1997), software verification (Chaki et al., 2004) and formal analysis of continuous (Tabuada & Pappas, 2004), hybrid (Tabuada et al., 2004) and discrete event systems (DESs). What makes bisimulation appealing is its capability in complexity mitigation and branching behavior preservation, specially when we deal with large scale distributed and concurrent systems such as multi-robot cooperative tasking, networked embedded systems, and traffic management.

Therefore, recent years have seen increasing research activities in employing bisimulation to DESs. References (Barrett & Lafortune, 1998), (Komenda & van Schuppen,

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2005) and (Su et al., 2010) used bisimulation for the control of deterministic systems subject to language equivalence. Madhusudan & Thiagarajan (2002) investigated the control for bisimulation equivalence with respect to a partial specification, in which the plant is taken to be deterministic and all events are treated to be controllable. Tabuada (2008) solved the controller synthesis problem for bisimulation equivalence in a wide variety of scenarios including continuous system, hybrid system and DESs, in which the bisimilarity controller is given as a morphism in the framework of category theory. Zhou et al. (2006) investigated the bisimilarity control for nondeterministic plants and nondeterministic specifications. A small model theorem was provided to show that a supervisor enforcing the bisimulation equivalence between the supervised system and the specification exists if and only if a state controllable automaton exists over the Cartesian product of the system and specification state spaces. This small model theorem was also extended for partial observation in (Zhou & Kumar, 2007). In both these works, the existence of a bisimilarity supervisor depends on the existence of a state controllable automaton, which is hard to calculate in a systematic way, and the complexity of checking the existence condition is doubly exponential. To reduce the computational complexity, Zhou & Kumar (2011) specialized to deterministic supervisors. The existence condition for a deterministic bisimilarity supervisor considering nondeterministic plants and nondeterministic specifications was identified. Moreover, the synthesis of deterministic supervisors, feasible superspecifications and infimal subspecifications were developed as well. Liu et al. (2011) introduced a simulation-based framework upon which the bisimilarity control for nondeterministic plants and nondeterministic specifications was studied. In particular, a new scheme based on the simulation relation was proposed for synchronization which is different from those commonly used synchronization operators such as parallel composition and product in the supervisory control literature.

This paper studies the supervisory control of nondeterministic plants for bisimulation equivalence with respect to deterministic specifications. Compared to the existing literature, the contributions of this paper mainly lie on the following aspects. First, a novel notion of synchronous simulation-based controllability is introduced as a necessary and sufficient condition for the existence of a bisimilarity enforcing supervisor. Although it is equivalent to the conditions in (Zhou & Kumar, 2011) specialized to deterministic specifications, it provides a great insight into what characters should a deterministic specification possesses for bisimilarity control. Second, a test algorithm is proposed to verify the existence condition, which is shown to be polynomial complexity (less than the complexity of the conditions in (Zhou & Kumar, 2011)). When the existence condi-

tion holds, we further present a systematic way to construct bisimilarity enforcing supervisors. Third, since a given specification does always guarantee the existence of a bisimilarity enforcing supervisor, a key question arises is how to find a maximal permissive specification which enables the synthesis of bisimilarity enforcing supervisors. To answer this question, we investigate the calculation of supremal synchronously simulation-based controllable sub-specifications by using two different methods. One is based on a recursive algorithm and the other directly computes such a sub-specification based on formulas.

The rest of this paper is organized as follows. Section 2 gives the preliminary and problem formulation. Section 3 presents the synthesis of bisimilarity enforcing supervisors. Section 4 investigates the test algorithm for the existence of a bisimilarity enforcing supervisor. Section 5 explores the calculation of maximal permissive sub-specifications. This paper concludes with section 6.

## 2. Preliminary and Problem Formulation

### 2.1. Preliminary Results

A DES is modeled as a nondeterministic automaton  $G = (X, \Sigma, x_0, \alpha, X_m)$ , where  $X$  is the set of states,  $\Sigma$  is the set of events,  $\alpha : X \times \Sigma \rightarrow 2^X$  is the transition function,  $x_0$  is the initial state and  $X_m \subseteq X$  is the set of marked states. The event set  $\Sigma$  can be partitioned into  $\Sigma = \Sigma_{uc} \cup \Sigma_c$ , where  $\Sigma_{uc}$  is the set of uncontrollable events and  $\Sigma_c$  is the set of controllable events. Let  $\Sigma^*$  be the set of all finite strings over  $\Sigma$  including the empty string  $\epsilon$ . The transition function  $\alpha$  can be extended from events to traces,  $\alpha : X \times \Sigma^* \rightarrow 2^X$ , which is defined inductively as: for any  $x \in X$ ,  $\alpha(x, \epsilon) = x$ ; for any  $s \in \Sigma^*$  and  $\sigma \in \Sigma$ ,  $\alpha(x, s\sigma) = \alpha(\alpha(x, s), \sigma)$ . If the transition function is a partial map  $\alpha : X \times \Sigma \rightarrow X$ ,  $G$  is said to be a deterministic automaton. For  $X_1 \subseteq X$ , the notation  $\alpha|_{X_1 \times \Sigma}$  means  $\alpha$  is restricted from a smaller domain  $X_1 \times \Sigma$  to  $2^{X_1}$ . Given  $X_1 \subseteq X$ , the subautomaton of  $G$  with respect to  $X_1$ , denoted by  $F_G(X_1)$ , is defined as:  $F_G(X_1) = (X_1, \Sigma, x_0, \alpha_1, X_{m1})$ , where  $\alpha_1 = \alpha|_{X_1 \times \Sigma}$  and  $X_{m1} = X_1 \cap X_m$ . The active event set at state  $x$  is defined as  $E_G(x) = \{\sigma \in \Sigma \mid \alpha(x, \sigma) \text{ is defined}\}$ . Given a string  $s \in \Sigma^*$ , the length of the string  $s$ , denoted as  $|s|$ , is the total numbers of events, and  $s(i)$  is the  $i$ -th event of this string, where  $1 \leq i \leq |s|$ . Given  $\Sigma_1 \subseteq \Sigma$ , a projection  $P_{\Sigma \rightarrow \Sigma_1} : \Sigma^* \rightarrow \Sigma_1^*$  is used to filter a string of events from  $\Sigma$  to  $\Sigma_1$ , and it is defined inductively as follows:  $P_{\Sigma \rightarrow \Sigma_1}(\epsilon) = \epsilon$ ; for any  $\sigma \in \Sigma$  and  $s \in \Sigma^*$ ,  $P_{\Sigma \rightarrow \Sigma_1}(s\sigma) = P_{\Sigma \rightarrow \Sigma_1}(s)\sigma$  if  $\sigma \in \Sigma_1$ , otherwise,  $P_{\Sigma \rightarrow \Sigma_1}(s\sigma) = P_{\Sigma \rightarrow \Sigma_1}(s)$ . The language generated by  $G$  is defined as  $L(G) = \{s \in \Sigma^* \mid \alpha(x_0, s) \text{ is defined}\}$ , and the marked language generated by  $G$  is defined as  $L_m(G) = \{s \in \Sigma^* \mid \alpha(x_0, s) \cap X_m \neq \emptyset\}$ . Consider three languages  $K, K_1, K_2 \subseteq \Sigma^*$ .

The Kleene closure of  $K$ , denoted as  $K^*$ , is the language  $K^* = \cup_{n \in \mathbb{N}} K^n$ , where  $K^0 = \{\epsilon\}$  and for any  $n \geq 0$ ,  $K^{n+1} = K^n K$ . The prefix closure of  $K$ , denoted as  $\overline{K}$ , is the language  $\overline{K} = \{s \in \Sigma^* \mid (\exists t \in \Sigma^*) st \in K\}$ . The quotient of  $K_1$  with respect to  $K_2$ , denoted as  $K_1/K_2$ , is the language  $K_1/K_2 = \{s \in \Sigma^* \mid (\exists t \in K_2) st \in K_1\}$ . For two languages  $K_1, K_2 \in \Sigma^*$  with  $K_2 \subseteq K_1 \neq \emptyset$ , let  $G_{(K_1, K_2)}$  be a deterministic automaton such that  $L(G_{(K_1, K_2)}) = K_1$  and  $L_m(G_{(K_1, K_2)}) = K_2$ . For a nondeterministic  $G$ , let  $det(G)$  be a minimal deterministic automaton such that  $L(det(G)) = L(G)$  and  $L_m(det(G)) = L_m(G)$ .

To model the interaction between automata, we introduce parallel composition as below (Cassandras & Lafortune, 2008).

**Definition 1.** Given  $G_1 = (X_1, \Sigma_1, x_{01}, \alpha_1, X_{m1})$  and  $G_2 = (X_2, \Sigma_2, x_{02}, \alpha_2, X_{m2})$ , the parallel composition of  $G_1$  and  $G_2$  is an automaton

$$G_1 \parallel G_2 = (X_1 \times X_2, \Sigma_1 \cup \Sigma_2, \alpha_{1 \parallel 2}, (x_{01}, x_{02}), X_{m1} \times X_{m2}),$$

where for any  $x_1 \in X_1$ ,  $x_2 \in X_2$  and  $\sigma \in \Sigma$ , the transition function is defined as:

$$\alpha_{1 \parallel 2}((x_1, x_2), \sigma) = \begin{cases} \alpha_1(x_1, \sigma) \times \alpha_2(x_2, \sigma) & \sigma \in E_{G_1}(x_1) \cap E_{G_2}(x_2); \\ \alpha_1(x_1, \sigma) \times \{x_2\} & \sigma \in E_{G_1}(x_1) \cap \sigma \in E_1 \setminus E_2; \\ \{x_1\} \times \alpha_2(x_2, \sigma) & \sigma \in E_{G_2}(x_2) \cap \sigma \in E_2 \setminus E_1; \\ \emptyset & otherwise. \end{cases}$$

When  $\Sigma_1 = \Sigma_2$ , parallel composition can be understood as a form of control, where a supervisor is designed to restrict the behavior of the plant.

Next we present the synchronized state map, which is used to find the synchronized state pairs of two automata (Zhou et al., 2006).

**Definition 2.** Given  $G_1 = (X_1, \Sigma_1, x_{01}, \alpha_1, X_{m1})$  and  $G_2 = (X_2, \Sigma_2, x_{02}, \alpha_2, X_{m2})$ , the synchronized state map  $X_{synG_1G_2}: X_1 \rightarrow 2^{X_2}$  from  $G_1$  to  $G_2$  is defined as

$$X_{synG_1G_2}(x_1) = \{x_2 \in X_2 \mid (\exists s \in \Sigma^*) x_1 \in \alpha_1(x_{01}, s) \wedge x_2 \in \alpha_2(x_{01}, s)\}.$$

Most literature on supervisory control aims to achieve language equivalence between the supervised system and the specification. The necessary and sufficient condition for the existence of a language enforcing supervisor is captured by the notion of language controllability as below (Ramadge & Wonham, 1987).

**Definition 3.** Given  $G = (X, \Sigma, x_0, \alpha, X_m)$ , a language  $K \subseteq L(G)$  is said to be language controllable with respect to  $L(G)$  and  $\Sigma_{uc}$  if

$$\overline{K}\Sigma_{uc} \cap L(G) \subseteq \overline{K}.$$

As a stronger behavior equivalence than language equivalence, bisimulation is stated as follows (Milner, 1989). It is known that bisimulation implies language equivalence and marked language equivalence, but the converse does not hold.

**Definition 4.** Given  $G_1 = (X_1, \Sigma, x_{01}, \alpha_1, X_{m1})$  and  $G_2 = (X_2, \Sigma, x_{02}, \alpha_2, X_{m2})$ , a simulation relation  $\phi$  is a binary relation  $\phi \subseteq X_1 \times X_2$  such that  $(x_1, x_2) \in \phi$  implies:

- (1)  $(\forall \sigma \in \Sigma)[\forall x'_1 \in \alpha_1(x_1, \sigma) \Rightarrow \exists x'_2 \in \alpha_2(x_2, \sigma) \text{ such that } (x'_1, x'_2) \in \phi];$
- (2)  $x_1 \in X_{m1} \Rightarrow x_2 \in X_{m2}.$

If there is a simulation relation  $\phi \subseteq X_1 \times X_2$  such that  $(x_{01}, x_{02}) \in \phi$ ,  $G_1$  is said to be simulated by  $G_2$ , denoted by  $G_1 <_\phi G_2$ . For  $\phi \subseteq (X_1 \cup X_2)^2$ , if  $G_1 <_\phi G_2$ ,  $G_2 <_\phi G_1$  and  $\phi$  is symmetric,  $\phi$  is called a bisimulation relation between  $G_1$  and  $G_2$ , denoted by  $G_1 \cong_\phi G_2$ . We sometimes omit the subscript  $\phi$  from  $<_\phi$  or  $\cong_\phi$  when it is clear from the context. Then we present a motivating example of this paper.

## 2.2. A Motivating Example

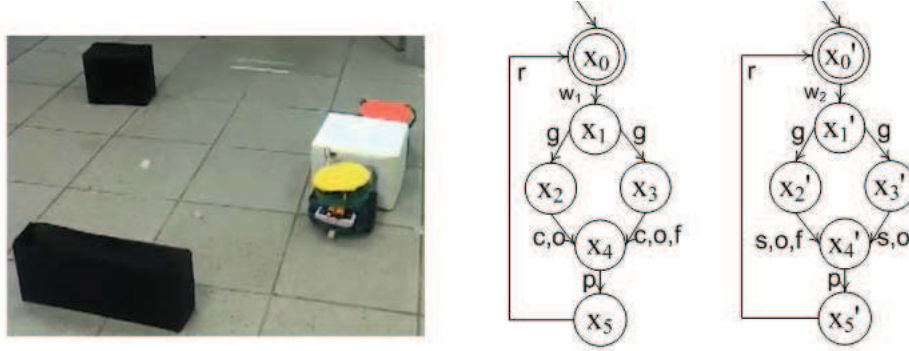


Figure 1: multi-robot system (MRS) (Left),  $G_1$  (Middle) and  $G_2$  (Right)

Consider a cooperative multi-robot system (MRS) configured in Fig. 1 (Left). The MRS consists of two robots  $R_1$  and  $R_2$ . Both of them have the same communication, position, pushing, scent-sensing and frequency-sensing capabilities.

Furthermore,  $R_1$  has color-sensing capabilities, while  $R_2$  has shape-sensing capability.  $R_1$  and  $R_2$  can cooperatively search and clear a dangerous object (the white cube) in the workspace. Initially,  $R_1$  and  $R_2$  are positioned outside the workspace. Let  $i = 1, 2$ . When the work request announces (event  $w_i$ ),  $R_i$  is required to enter the workspace. Due to actuator limitations, it nondeterministically goes along one of two pre-defined paths (event  $g$ ). In the first path,  $R_1$  activates color-sensing (event  $c$ ) and scent-sensing (event  $o$ ) capabilities to detect the dangerous object; whereas in the second path, besides color-sensing and scent-sensing capabilities,  $R_1$  also activates frequency-sensing (event  $f$ ) for detection. Similarly,  $R_2$  activates shape-sensing (event  $s$ ), scent-sensing and frequency-sensing capabilities in the first path, while in the second path it activates shape-sensing and scent-sensing capabilities. After detecting the dangerous object,  $R_i$  pushes the dangerous object outward the workspace (event  $p$ ), and then returns to the initial position (event  $r$ ) for the next implementation.

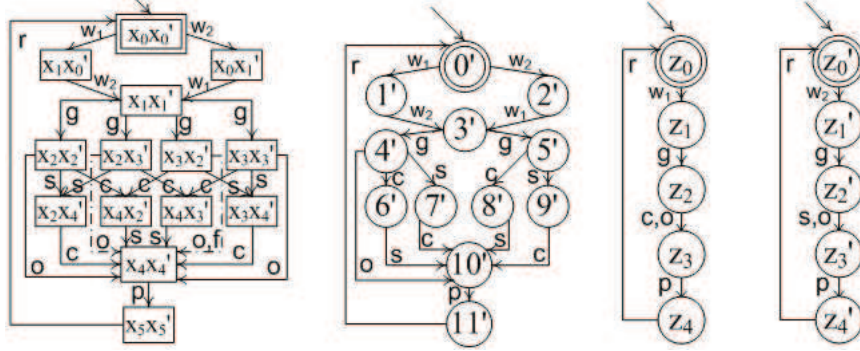


Figure 2:  $G_1 || G_2$  (First Left),  $R$  (Second Left),  $S_1$  (Second Right) and  $S_2$  (First Right)

The automaton model  $G_i$  of  $R_i$  with alphabet  $\Sigma_i$  is shown in Fig. 1, where  $\Sigma_1 = \{w_1, g, c, o, f, p, r\}$  and  $\Sigma_2 = \{w_2, g, s, o, f, p, r\}$ . Since  $R_i$  can not disable the host computer to broadcast the work announcement, the event  $w_i$  is deemed uncontrollable, that is  $w_i \in \Sigma_{uci}$ . The rest events are controllable. The cooperative behavior of  $R_1$  and  $R_2$  can be represented as  $G_1 || G_2$  (Fig. 2 (First Left)). The specification  $R$ , configured in Fig. 2, is given in order to restrict the cooperative behavior  $G_1 || G_2$ . According to the specification, after both  $R_1$  and  $R_2$  receive the work command and go to the workspace, two possible states may be reached by the MRS nondeterministically. In the first state, the color sensor, the shape sensor and the scent sensors can be adopted to confirm an objective is dangerous.

However, to save the energy, in the second state only the color sensor and the shape sensor can be adopted for dangerous object detection. After the detection, the dangerous object is cleared from the workspace.

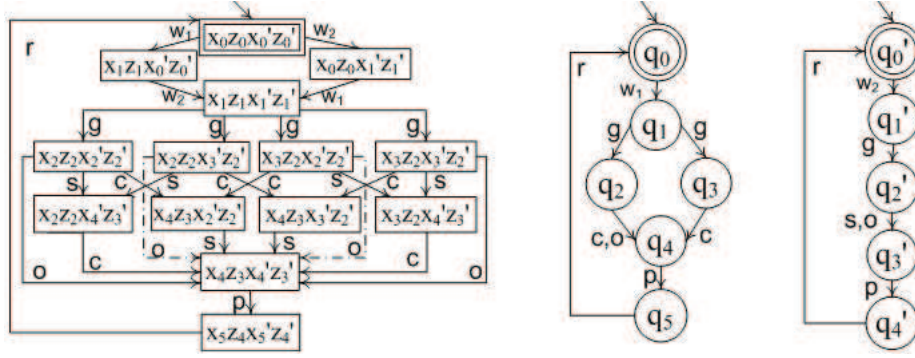


Figure 3:  $\|_{i \in \{1,2\}} G_i \| S_i$  (Left),  $R_{s_1}$  (Middle) and  $R_{s_2}$  (Right)

For such a MRS, if we use language equivalence as behavior equivalence, the control target is to design supervisors  $S_1$  and  $S_2$  such that  $L(\|_{i \in \{1,2\}} G_i \| S_i) = L(R)$ . According to the results in (Willner & Heymann, 1991), this problem can be solved by designing  $S_i$  such that  $L(G_i \| S_i) = P_{\Sigma_1 \cup \Sigma_2 \rightarrow \Sigma_i}(L(R))$ . Since  $P_{\Sigma_1 \cup \Sigma_2 \rightarrow \Sigma_i}(L(R))$  is language controllable with respect to  $L(G_i)$  and  $\Sigma_{uci}$ , we can construct  $S_i$  as shown in Fig. 2. So the supervised system  $\|_{i \in \{1,2\}} G_i \| S_i$  (Fig. 3 (Left)) is language equivalent to  $L(R)$ . However, it can be seen that  $\|_{i \in \{1,2\}} G_i \| S_i$  enables all the color sensor, the shape sensor and the scent sensors for dangerous object detection, which violates the energy saving requirement in the specification. Hence language equivalence is not adequate for this case, which calls for the use of bisimulation as behavior equivalence. That is, we need design supervisor  $S'_i$  such that  $\|_{i \in \{1,2\}} G_i \| S'_i \cong R$ . For such a bisimilarity control problem, a promising method (Karimadini & Lin, 2011) is to decompose the global specification  $R$  into sub-specifications  $R_{s_i}$  with alphabet  $\Sigma_i$  for  $R_i$  (Fig. 3) such that  $\|_{i \in \{1,2\}} R_{s_i} \cong R$ . If we can design  $S'_i$  such that  $G_i \| S'_i \cong R_{s_i}$ , then  $\|_{i \in \{1,2\}} G_i \| S'_i \cong R$ . In particular,  $R_{s_2}$  is deterministic, which motivates us to consider the bisimilarity control for deterministic specifications in this paper.

### 2.3. Problem Formulation

In the rest of paper, unless otherwise stated we will use  $G = (X, \Sigma, \alpha, x_0, X_m)$ ,  $R = (Q, \Sigma, \delta, q_0, Q_m)$  and  $S = (Y, \Sigma, \beta, y_0, Y_m)$  to denote the nondeterministic plant,

the deterministic specification and the supervisor (possibly nondeterministic) respectively. Next we formalize the notion of bisimilarity enforcing supervisor, which always enables all uncontrollable events and enforces bisimilarity between the supervised system and the specification.

**Definition 5.** Given a plant  $G$  and a specification  $R$ , a supervisor  $S$  is said to be a bisimilarity enforcing supervisor for  $G$  and  $R$  if:

- (1) There is a bisimulation relation  $\phi$  such that  $G||S \cong_{\phi} R$ ;
- (2)  $(\forall y \in Y)(\forall \sigma \in \Sigma_{uc}) \beta(y, \sigma) \neq \emptyset$ .

This paper aims to solve the following problems.

**Problem 1:** Given a nondeterministic plant  $G$  and a deterministic specification  $R$ , what condition guarantees the existence of a bisimilarity enforcing supervisor  $S$  for  $G$  and  $R$ ?

**Problem 2:** How to check this condition effectively?

**Problem 3:** If the condition is satisfied, how to construct a bisimilarity enforcing supervisor  $S$ ?

**Problem 4:** If the condition is not satisfied, how to obtain a maximal permissive sub-specification which enables the synthesis of bisimilarity enforcing supervisors?

### 3. Supervisory Control for Bisimilarity

This section investigates Problem 1 and Problem 3, also called the bisimilarity enforcing supervisor synthesis problem. We begin with the existence condition of a bisimilarity enforcing supervisor. For sufficiency, since we need design a bisimilarity enforcing supervisor, the following concept is introduced.

**Definition 6.** Given  $G_1 = (X_1, \Sigma, x_{01}, \alpha_1, X_{m1})$ , the uncontrollable augment automaton  $G_{1uc}$  of  $G_1$  is defined as:

$$G_{1uc} = (X_1 \cup \{D_d\}, \Sigma, x_{01}, \alpha_{uc}, X_{m1}),$$

where for any  $x \in X_1 \cup \{D_d\}$  and  $\sigma \in \Sigma$ :

$$\alpha_{uc}(x, \sigma) = \begin{cases} \alpha_1(x, \sigma) & \sigma \in E_{G_1}(x); \\ \{D_d\} & (\sigma \in \Sigma_{uc} \setminus E_{G_1}(x)) \vee (x = D_d \wedge \sigma \in \Sigma_{uc}); \\ \emptyset & \text{otherwise.} \end{cases}$$

We can see that an uncontrollable augment automaton can be employed in the construction of bisimilarity enforcing supervisors because it naturally satisfies the condition (2) required for a bisimilarity enforcing supervisor (Definition 5).

On the other side, for necessity we have  $G||S \cong R$ , which implies  $R < G||S < G$ . Hence  $R < G$  is a necessary condition to guarantee the existence of a bisimilarity enforcing supervisor. Moreover,  $G||S \cong R$  implies  $L(G||S) = L(R)$ , thus language controllability of the specification is also a necessary condition for the existence of a bisimilarity enforcing supervisor. To satisfy those necessary conditions, we will introduce synchronous simulation-based controllability as a property of the specification. Before that, we need the following concept.

**Definition 7.** Given  $G_1 = (X_1, \Sigma, x_{01}, \alpha_1, X_{m1})$ ,  $G_2 = (X_2, \Sigma, x_{02}, \alpha_2, X_{m2})$  and a simulation relation  $\phi$  such that  $G_1 <_\phi G_2$ ,  $\phi$  is called a synchronous simulation relation from  $G_1$  to  $G_2$  if  $(x_1, x_2) \in \phi$  for any  $x_1 \in X_1$  and  $x_2 \in X_{\text{syn}G_1G_2}(x_1)$ .

If there exists a synchronous simulation relation  $\phi$  from  $G_1$  to  $G_2$ ,  $G_1$  is said to be synchronously simulated by  $G_2$ , denoted as  $G_1 <_{\text{syn}\phi} G_2$ . For a deterministic specification  $R$ , if  $R$  is synchronously simulated by  $G$ , then  $G$  possesses the branches which are bisimilar to  $R$  and the branches which are outside  $L(R)$ . Hence it turns out that  $G||R \cong R$ . If  $R$  is further language controllable with respect to  $L(G)$  and  $\Sigma_{uc}$ , then  $G||R = G||R_{uc}$ , implying that  $R_{uc}$  is a candidate of bisimilarity enforcing supervisor. Base on this observation, we provide the following concept.

**Definition 8.** Given  $G_1 = (X_1, \Sigma, x_{01}, \alpha_1, X_{m1})$  and  $G_2 = (X_2, \Sigma, x_{02}, \alpha_2, X_{m2})$ ,  $G_1$  is said to be synchronously simulation-based controllable with respect to  $G_2$  and  $\Sigma_{uc}$  if it satisfies:

- (1) There is a synchronous simulation relation  $\phi$  such that  $G_1 <_{\text{syn}\phi} G_2$ ;
- (2)  $L(G_1)$  is language controllable with respect to  $L(G_2)$  and  $\Sigma_{uc}$ .

It is immediate to see that when  $R$  is synchronously simulation-based controllable with respect to  $G$  and  $\Sigma_{uc}$ , it not only satisfies the necessary conditions ( $R < G$  and language controllability of  $L(R)$ ) for the existence of a bisimilarity enforcing supervisor but also enables the development of  $R_{uc}$  as a bisimilarity enforcing supervisor to accomplish the sufficiency of the existence condition.

Then we present a necessary and sufficient condition for the existence of a bisimilarity enforcing supervisor.

**Theorem 1.** *Given a plant  $G$  and a deterministic specification  $R$ , there exists a bisimilarity enforcing supervisor  $S$  for  $G$  and  $R$  if and only if  $R$  is synchronously simulation-based controllable with respect to  $G$  and  $\Sigma_{uc}$ .*

PROOF. For sufficiency, we choose  $R_{uc}$  as the supervisor. Let  $G\|R = (X_{\parallel}, \Sigma, (x_0, q_0), \alpha_{\parallel}, X_{m\parallel})$ . Consider a relation  $\phi_1 = \{((x, q), q) \mid (x, q) \in X_{\parallel}\}$ . We show that  $\phi_1 \cup \phi_1^{-1}$  is a bisimulation relation from  $G\|R$  to  $R$ . First note that  $((x_0, q_0), q_0) \in \phi_1$ . Pick  $((x, q), q) \in \phi_1$  and  $(x', q') \in \alpha_{\parallel}((x, q), \sigma)$ , where  $\sigma \in \Sigma$ . By the definition of parallel composition, we have  $q' \in \delta(q, \sigma)$ , which implies  $((x', q'), q') \in \phi_1$ . When  $(x', q') \in X_{m\parallel}$ , then  $q' \in Q_m$ . On the other side, pick  $(q, (x, q)) \in \phi_1^{-1}$  and  $q' \in \delta(q, \sigma)$ . Since  $(x, q) \in X_{\parallel}$  and there is a synchronous simulation relation  $\phi$  such that  $R <_{\text{syn}\phi} G$ , we have  $(q, x) \in \phi$ . Then there is  $x' \in \alpha(x, \sigma)$  such that  $(q', x') \in \phi$ , and if  $q' \in Q_m$ , then  $x' \in X_m$ . It follows that  $(x', q') \in \alpha_{\parallel}((x, q), \sigma)$  and  $(x', q') \in X_{m\parallel}$  when  $q' \in Q_m$ . That is,  $(q', (x', q')) \in \phi_1^{-1}$ . Hence  $G\|R \cong_{\phi_1 \cup \phi_1^{-1}} R$ . Moreover from determinism and language controllability of  $R$  and the fact that  $R_{uc}$  adds every state a transition to  $D_d$  through undefined uncontrollable events does not change the result of parallel composition, we have  $G\|R_{uc} = G\|R$ . It implies that  $G\|R_{uc} \cong_{\phi_1 \cup \phi_1^{-1}} R$ .

For necessity, suppose there is a bisimilarity enforcing supervisor  $S$  for  $G$  and  $R$ . Then, there is a bisimulation relation  $\phi' = \phi \cup \phi^{-1}$  such that  $R <_{\phi} G\|S$  and  $G\|S <_{\phi^{-1}} R$ . Let  $G\|S = (X_{G\|S}, \Sigma, (x_0, y_0), \alpha_{G\|S}, X_{mG\|S})$ . Consider a relation  $\phi_1 = \{(q, x) \in Q \times X \mid (\exists y \in Y) (q, (x, y)) \in \phi\}$ . We show that  $\phi_1$  is a synchronous simulation relation from  $R$  to  $G$ . By the definition of parallel composition,  $\phi_1$  is a simulation relation from  $R$  to  $G$ . Assume there is  $q \in Q$  and  $x' \in X_{\text{syn}RG}(q)$  such that  $(q, x') \notin \phi_1$ . Hence there exists  $s \in \Sigma^*$  such that  $q \in \delta(q_0, s)$  and  $x' \in \alpha(x_0, s)$ . Since  $R <_{\phi} G\|S$ , for  $q \in \delta(q_0, s)$ , there is  $(x, y) \in \alpha_{G\|S}((x_0, y_0), s)$  such that  $(q, (x, y)) \in \phi$ , which implies  $y \in \beta(y_0, s)$  and in turn implies  $(x', y) \in \alpha_{G\|S}((x_0, y_0), s)$ . Because  $G\|S <_{\phi^{-1}} R$ , for  $(x', y) \in \alpha_{G\|S}((x_0, y_0), s)$ , there is  $q' \in \delta(q_0, s)$  such that  $((x', y), q') \in \phi^{-1}$ . Since  $R$  is deterministic, we have  $q = q'$ . Therefore,  $(q, (x', y)) \in \phi$ , which implies  $(q, x') \in \phi_1$ . It introduces a contradiction. Then the assumption is not correct. That is, for any  $q \in Q$  and  $x \in X_{\text{syn}RG}(q)$ ,  $(q, x) \in \phi_1$ . So  $R <_{\text{syn}\phi_1} G$ . Next we show language controllability of  $L(R)$ . Since a bisimilarity enforcing supervisor  $S$  enables all uncontrollable events at each state,  $L(G\|S)$  is language controllable with respect to  $L(G)$  and  $\Sigma_{uc}$ , further,  $G\|S \cong R$  implies  $L(G\|S) = L(R)$ . It follows that  $L(R)$  is language controllable w.r.t.  $L(G)$  and  $\Sigma_{uc}$ . So  $R$  is synchronously simulation-based controllable w.r.t.  $G$  and  $\Sigma_{uc}$ .

**Remark 1.** Theorem 1 shows that if a deterministic  $R$  is synchronously simulation-based controllable with respect to  $G$  and  $\Sigma_{uc}$ ,  $R_{uc}$  is a bisimilarity enforcing supervisor for  $G$  and  $R$ . Here synchronous simulation-based controllability of  $R$  is equivalent to the conditions ( $G\|\text{det}(R) \cong R$  and language controllability of  $L(R)$ ) specialized to deterministic specifications (Zhou & Kumar, 2011) to ensure

the existence of a deterministic bisimilarity supervisor. However, the notion of synchronous simulation-based controllability offers computation advantages compared to the conditions in (Zhou & Kumar, 2011) (See section 4). Moreover, it enables the calculation of maximal permissive sub-specification when the existence condition for a bisimilarity enforcing supervisor does not hold (See section 5).

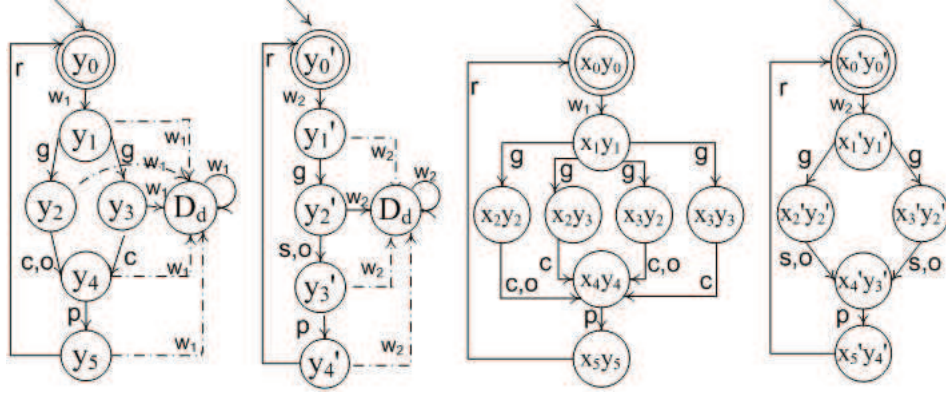


Figure 4:  $S'_1$  (First Left),  $S'_2$  (Second Left),  $G_1||S'_1$  (Second Right) and  $G_2||S'_2$  (First Right)

Now we revisit the motivating example.

**Example 1.** Let  $i=1, 2$ . We need design supervisor  $S'_i$  such that  $G_i||S'_i \cong R_{s_i}$ . Since  $R_{s_2}$  is deterministic and synchronously simulation-based controllable with respect to  $G_2$  and  $\Sigma_{uc2}=\{w_2\}$ , from Theorem 1 we can design  $(R_{s_2})_{uc}$  to be  $S'_2$  (Fig. 4 (Second Left)). The supervised system  $G_2||S'_2$  is shown in Fig. 4 (First Right) and it can be seen that  $G_2||S'_2 \cong_{\phi \cup \phi^{-1}} R_{s_2}$ , where  $\phi=\{(q'_0, (x'_0, y'_0)), (q'_1, (x'_1, y'_1)), (q'_2, (x'_2, y'_2)), (q'_2, (x'_3, y'_2)), (q'_3, (x'_4, y'_3)), (q'_4, (x'_5, y'_4))\}$ . In addition,  $S'_1$  for  $G_1$  can be designed as shown in Fig. 4 (First Left) according to our results in (Sun & Lin, 2012). Then  $G_1||S'_1 \cong R_{s_1}$  (Fig. 4 (Second Right)). As a result,  $\|_{i \in \{1,2\}} G_i||S'_i \cong R$ .

#### 4. A Test Algorithm for the Existence of a Bisimilarity Enforcing Supervisor

To solve Problem 2, an algorithm is proposed in this section to test the existence of a bisimilarity enforcing supervisor. We start by introducing synchronously simulation-based controllable product, which will be used in the test algorithm.

**Definition 9.** Given  $G_1 = (X_1, \Sigma, x_{01}, \alpha_1, X_{m1})$  and  $G_2 = (X_2, \Sigma, x_{02}, \alpha_2, X_{m2})$ , the synchronously simulation-based controllable product of  $G_1$  and  $G_2$  is an automaton

$$G_1 \parallel_{\text{sync}} G_2 = ((X_1 \times X_2) \cup \{q_d, q'_d\}, \Sigma, \alpha_{12}, (x_{01}, x_{02}), X_{m1} \times X_{m2}),$$

where for any  $(x_1, x_2) \in X_1 \times X_2$  and  $\sigma \in \Sigma$ , the transition function is defined as:

$$\alpha_{12}((x_1, x_2), \sigma) = \begin{cases} \alpha_1(x_1, \sigma) \times \alpha_2(x_2, \sigma) & \sigma \in E_{G_1}(x_1) \cap E_{G_2}(x_2); \\ q_d & \sigma \in E_{G_1}(x_1) \setminus E_{G_2}(x_2); \\ q'_d & \sigma \in \Sigma_{uc} \cap (E_{G_2}(x_2) \setminus E_{G_1}(x_1)); \\ \emptyset & \text{otherwise.} \end{cases}$$

Since synchronous simulation-based controllability is a necessary and sufficient condition for the existence of a bisimilarity enforcing supervisor, the following algorithm for testing synchronous simulation-based controllability of  $R$  also verifies the existence of a bisimilarity enforcing supervisor for  $G$  and  $R$ .

**Algorithm 1.** *Given a plant  $G$  and a deterministic specification  $R$ , the algorithm for testing synchronous simulation-based controllability of  $R$  with respect to  $G$  and  $\Sigma_{uc}$  is described as below.*

*Step 1: Obtain  $R \parallel_{\text{sync}} G = (X_{\text{sync}}, \Sigma, \alpha_{\text{sync}}, (q_0, x_0), X_{m\text{sync}})$ ;*

*Step 2:  $R$  is synchronously simulated-based controllable with respect to  $G$  and  $\Sigma_{uc}$  if and only if  $q_d$  and  $q'_d$  are not reachable in  $R \parallel_{\text{sync}} G$  and  $x \in X_m$  for any reachable state  $(q, x)$  in  $R \parallel_{\text{sync}} G$  with  $q \in Q_m$ .*

**Theorem 2.** *Algorithm 1 is correct.*

**PROOF.** From the definition of synchronously simulation-based controllable product, it is obvious that any  $(q, x)$  satisfying  $x \in X_{\text{sync}RG}(q)$  is a state reachable in  $R \parallel_{\text{sync}} G$ , and any  $(q, x) \in X_{\text{sync}} \setminus \{q_d, q'_d\}$  satisfies that  $x \in X_{\text{sync}RG}(q)$ . For synchronous simulation-based controllability to hold, condition (1) and condition (2) of Definition 8 should be satisfied. On the other hand, if condition (1) is violated, there are two cases. Case 1: there exist  $(q, x)$  and  $\sigma \in \Sigma$  such that  $x \in X_{\text{sync}RG}(q)$  and  $\sigma \in E_R(q) \setminus E_G(x)$ . So  $q_d \in \alpha_{\text{sync}}((q, x), \sigma)$ . Case 2: there is  $(q, x)$  such that  $x \in X_{\text{sync}RG}(q)$  and  $x \notin X_m$  when  $q \in Q_m$ . If condition (2) is violated, i.e. there exist  $(q, x)$  and  $\sigma \in \Sigma_{uc}$  such that  $x \in X_{\text{sync}RG}(q)$  and  $\sigma \in E_G(x) \setminus E_R(q)$ . So  $q'_d \in \alpha_{\text{sync}}((q, x), \sigma)$ . It follows that  $q_d$  and  $q'_d$  are reachable in  $R \parallel_{\text{sync}} G$  or  $x \notin X_m$  for any reachable state  $(q, x)$  in  $R \parallel_{\text{sync}} G$  with  $q \in Q_m$  iff  $R$  is not synchronously simulated-based controllable w.r.t.  $G$  and  $\Sigma_{uc}$ .

**Remark 2.** Algorithm 1 can be terminated because the state sets and the event sets of  $R$  and  $G$  are finite. Since  $G$  is nondeterministic and  $R$  is deterministic, their numbers of transitions are  $O(|X|^2|\Sigma|)$  and  $O(|Q||\Sigma|)$  respectively. Then the complexity of constructing  $R||_{\text{sync}}G$  is  $O(|X|^2|Q|^2|\Sigma|)$ . In addition, the complexity of checking the reachability of  $q_d$  and  $q'_d$  in  $R||_{\text{sync}}G$  is  $O(\log(|X||Q|))$  (Jones, 1975). So the complexity of Algorithm 1 is  $O(|X|^2|Q|^2|\Sigma|)$ . That is, the algorithm for testing the existence of a bisimilarity enforcing supervisor has polynomial complexity. Zhou & Kumar (2011) used the conditions such as  $G||_{\text{det}}(R) \cong R$  and  $L(R)$  is language controllable with respect to  $L(G)$  and  $\Sigma_{uc}$  to guarantee the existence of a deterministic supervisor that achieves bisimulation equivalence. The complexity of verifying those conditions with respect to deterministic specifications is  $O(|X|^2|Q|^2|\Sigma|^3 \log(|X||Q|))$  (Remark 2 in (Zhou & Kumar, 2011)). Hence, we argue that Algorithm 1 is more effective.

We provide the following example to illustrate the algorithm for checking synchronous simulation-based controllability.

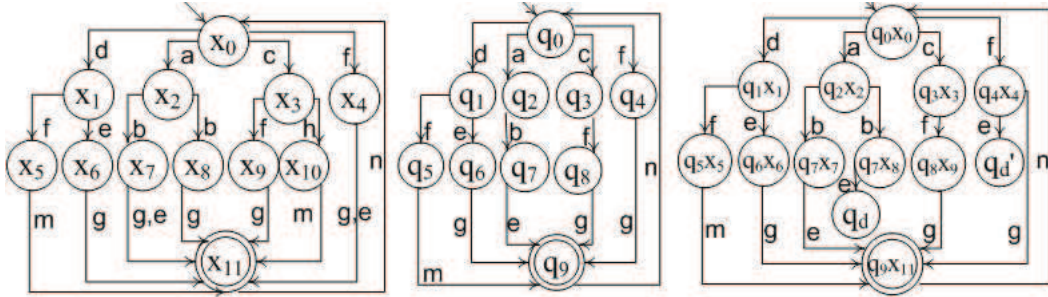


Figure 5: Plant  $G$  (Left), Specification  $R$  (Middle) and  $R||_{\text{sync}}G$  (Right) of Example 2

**Example 2.** Consider a plant  $G$  and a specification  $R$  with  $\Sigma_{uc} = \{b, e\}$  configured in Fig. 5. We can see that  $R$  is not synchronously simulation-based controllable with respect to  $G$  and  $\Sigma_{uc}$  because for  $f \in L(G) \cap L(R)$  and  $e \in \Sigma_{uc}$ ,  $fe \in L(G) \setminus L(R)$ , and  $e$  is defined at  $q_7$  but not  $x_8 \in X_{\text{synRG}}(q_7)$ .

Next we use Algorithm 1 to test synchronously simulation-based controllability of  $R$ . The synchronously simulation-based controllable product  $R||_{\text{sync}}G$  is shown in Fig. 5 (Right). It can be seen that  $q_d$  and  $q'_d$  are reachable in  $R||_{\text{sync}}G$ . Hence  $R$  is not synchronously simulation-based controllable with respect to  $G$  and  $\Sigma_{uc}$ .

## 5. Supremal Synchronously Simulation-Based Controllable Sub-specifications

This section studies Problem 4, i.e., the synthesis of supremal synchronously simulation-based controllable sub-specifications, because a synchronous simulation-based controllable sub-specification ensures the existence of a bisimilarity enforcing supervisor. First we introduce the notion of supremal.

Given  $(A, \leq)$  and  $A' \subseteq A$ , where  $\leq \subseteq A \times A$  is a transitive and reflexive relation over  $A$ ,  $x \in A$  is said to be a supremal of  $A'$ , denoted by  $\sup A'$ , if it satisfies:

- (1)  $\forall y \in A': y \leq x$ ;
- (2)  $\forall z \in A : [\forall y \in A' : y \leq z] \Rightarrow [x \leq z]$ .

When we define the supremal of  $A'$ , a set  $(A, \leq)$  should be given with respect to the element of  $A'$ . If the elements of  $A'$  are languages, the set  $(2^{\Sigma^*}, \subseteq)$  should be applied because  $2^{\Sigma^*}$  includes all languages over alphabet  $\Sigma$  and language inclusion fully captures the comparison between two languages. However, if the elements of  $A'$  are automata, the set  $(B, <)$  should be applied, where  $B$  is a full set of automata with alphabet  $\Sigma$  and  $< \subseteq B \times B$  is the simulation relation, since  $B$  includes all automata over alphabet  $\Sigma$  and the simulation relation is adequate for automata (possibly nondeterministic) comparison.

We consider the class of sub-specifications that satisfies synchronous simulation-based controllability as below.

$$C_1 := \{R' \mid R' \text{ is deterministic, } R' < R \text{ and } R' \text{ is synchronous simulation-based controllable w.r.t. } G \text{ and } \Sigma_{uc}\}$$

It can be seen that the supremal of  $C_1$  with respect to  $(B, <)$  is a supremal synchronously simulation-based controllable sub-specification. However, it is difficult to directly calculate the supremal of  $C_1$  because  $C_1$  is not closed under the upper bound (join) operator with respect to  $(B, <)$  (Zhou & Kumar, 2011). To encounter this problem, we would like to convert the automaton set  $C_1$  into equivalently expressed language sets which are closed under the upper bound (set union) operator with respect to  $(2^{\Sigma^*}, \subseteq)$  (Cassandras & Lafortune, 2008). Next we do this conversion item by item. First, for two deterministic automata  $R'$  and  $R$ , the condition  $R' < R$  is equivalent to the language condition  $L(R') \subseteq L(R)$  and  $L_m(R') \subseteq L_m(R)$ . Second, language controllability required in synchronous simulation-based controllability is naturally a language description. It remains to convert synchronous simulation relation required in synchronous simulation-based controllability to an equivalent language condition. To complete the conversion, we need the following concept.

**Definition 10.** Given  $G = (X, \Sigma, x_0, \alpha, X_m)$ , the synchronous state merger operator on  $G$  is defined as an automaton

$$F_{syn}(G) = (X_{syn}, \Sigma, \{x_0\}, \alpha_{syn}, X_{msyn}),$$

where  $X_{syn} = 2^X$ ,  $X_{msyn} = \{Y_1 \mid Y_1 \subseteq X_m\}$ , and for any  $A \in X_{syn}$  and  $\sigma \in \Sigma$ , the transition function is defined as:

$$\alpha_{syn}(A, \sigma) = \begin{cases} \cup_{x \in A} \alpha(x, \sigma) & \sigma \in \cap_{x \in A} E_G(x); \\ \text{undefined} & \text{otherwise.} \end{cases}$$

By using  $F_{syn}(G)$ , the synchronous simulation relation from a deterministic automaton  $G_1$  to a plant  $G$  is equivalent to language conditions  $L(G_1) \subseteq L(F_{syn}(G))$  and  $L_m(G_1) \subseteq L_m(F_{syn}(G))$ , which is illustrated by the following proposition.

**Proposition 1.** *Given a plant  $G$  and a deterministic automaton  $G_1$ , there is a synchronous simulation relation  $\phi$  such that  $G_1 \prec_{syn\phi} G$  iff  $L(G_1) \subseteq L(F_{syn}(G))$  and  $L_m(G_1) \subseteq L_m(F_{syn}(G))$ .*

PROOF. Let  $F_{syn}(G) = (X_f, \Sigma, \{x_0\}, \alpha_f, X_{mf})$ ,  $G_1 = (X_1, \Sigma, x_{01}, \alpha_1, X_{m1})$  and  $G_L = G_1 \parallel G = (X_L, \Sigma, (x_{01}, x_0), \alpha_L, X_{mL})$ . For sufficiency, consider a relation  $\phi = \{(x_1, x) \in X_1 \times X \mid x \in X_{synG_1G}(x_1)\}$ . We show that  $\phi$  is a synchronous simulation relation from  $G_1$  to  $G$ . First note that  $(x_{01}, x_0) \in \phi$ . Pick  $(x_1, x) \in \phi$  and  $x'_1 \in \alpha_1(x_1, \sigma)$ , where  $\sigma \in \Sigma$ . Since  $x \in X_{synG_1G}(x_1)$ , there is  $s \in \Sigma^*$  such that  $x_1 \in \alpha_1(x_{01}, s)$  and  $x \in \alpha(x_0, s)$ . Hence  $s, s\sigma \in L(G_1)$ , moreover,  $L(G_1) \subseteq L(F_{syn}(G))$ . It follows that  $s, s\sigma \in L(F_{syn}(G))$ . Therefore there exist  $A = \alpha_f(\{x_0\}, s)$  and  $A_1 = \alpha_f(A, \sigma)$ . By the definition of  $F_{syn}(G)$ , we have  $x \in A$  and  $\sigma \in \cap_{x'' \in A} E_G(x'')$ , which implies there is  $x' \in \alpha(x, \sigma)$  such that  $x' \in X_{synG_1G}(x'_1)$ , i.e.  $(x'_1, x') \in \phi$ . Next we show that  $x_1 \in X_{m1}$  implies  $x \in X_m$ . Because  $x_1 \in X_{m1}$ , we have  $s \in L_m(G_1)$ , in addition,  $L_m(G_1) \subseteq L_m(F_{syn}(G))$ . It follows  $s \in L_m(F_{syn}(G))$ , that is  $A \subseteq X_m$ , implying  $x \in X_m$ . So  $G_1 \prec_{syn\phi} G$ .

For necessity, the induction method is used to prove  $s \in L(F_{syn}(G))$  for any  $s \in L(G_1)$ , that is  $L(G_1) \subseteq L(F_{syn}(G))$ . (1)  $|s| = 0$ , then  $s = \epsilon$ . It is obvious that  $\epsilon \in L(F_{syn}(G))$ . (2) Assume when  $|s| = n$ , we have  $s \in L(F_{syn}(G))$  for any  $s \in L(G_1)$ . (3)  $|s| = n + 1$ . Let  $s = s_1\sigma$ , where  $\sigma \in \Sigma$ . Because  $s_1\sigma \in L(G_1)$  and  $G_1$  is deterministic, for any  $x_2 \in \alpha_1(x_{01}, s_1)$ , we have  $\sigma \in E_{G_1}(x_2)$ . Since  $G_1 \prec_{syn\phi} G$ , for any  $x'' \in \alpha(x_0, s_1)$ , we have  $(x_2, x'') \in \phi$ . It follows that  $\sigma \in \cap_{x'' \in \alpha(x_0, s_1)} E_G(x'')$ . In addition,  $|s_1| = n$  implies  $s_1 \in L(F_{syn}(G))$ , which in turn implies there is  $A_1 = \alpha_f(\{x_0\}, s_1)$  such that  $x'' \in A_1$ . Hence  $A_2 = \alpha_f(A_1, \sigma) =$

$\cup_{x'' \in A_1} \alpha(x'', \sigma)$ , that is,  $s_1 \sigma \in L(F_{syn}(G))$ . Therefore for any  $s \in L(G_1)$ , we have  $s \in L(F_{syn}(G))$ , i.e.  $L(G_1) \subseteq L(F_{syn}(G))$ . Next we show  $L_m(G_1) \subseteq L_m(F_{syn}(G))$  by proving  $s' \in L_m(F_{syn}(G))$  for any  $s' \in L_m(G_1)$ . Since  $s' \in L_m(G_1)$ , there is  $x_4 \in \alpha_1(x_{01}, s')$  such that  $x_4 \in X_{m1}$ . Because  $G_1 <_{syn\phi} G$  implies  $(x_4, x''') \in \phi$  for any  $x''' \in \alpha(x_0, s')$ , we have  $x''' \in X_m$ . Definition of  $F_{syn}(G)$  implies  $s' \in L_m(F_{syn}(G))$ , i.e.  $L_m(G_1) \subseteq L_m(F_{syn}(G))$ .

Hence the automaton set  $C_1$  can be converted into the following language sets:

$$\begin{aligned} C_2 &:= \{L_1 \subseteq L(R) \cap L(F_{syn}(G)) \mid L_1 = \overline{L_1} \text{ and } L_1 \text{ is language controllable} \\ &\quad \text{w.r.t. } L(G) \text{ and } \Sigma_{uc}\}; \\ C_3 &:= \{L_1 \cap L_m(R) \cap L_m(F_{syn}(G)) \mid L_1 \in C_2\}. \end{aligned}$$

The computation of supremal synchronously simulation-based controllable sub-specification, i.e.,  $supC_1$ , with respect to  $(B, <)$ , can be achieved through the computation of the supremal languages of  $C_2$  and  $C_3$  with respect to  $(2^{\Sigma^*}, \subseteq)$  as shown in the following theorem.

**Theorem 3.** *Given a plant  $G$  and a deterministic specification  $R$ , if  $supC_2 \neq \emptyset$ , then  $G_{(supC_2, supC_3)} \in supC_1$ .*

PROOF. Let  $L_1 = supC_2 \neq \emptyset$  and  $L'_1 = supC_2 \cap L_m(R) \cap L_m(F_{syn}(G)) = supC_3$ . First we show that  $G_{(L_1, L'_1)} \in C_1$ . Since  $L_1 = supC_2$ , we have  $L_1 \in C_2$ , which implies  $L_1$  is language controllable w.r.t.  $L(G)$  and  $\Sigma_{uc}$  and  $L_1 \subseteq L(F_{syn}(G))$ . In addition, definition of  $L'_1$  implies  $L'_1 \subseteq L_m(F_{syn}(G))$ . From Proposition 1, it follows that  $G_{(L_1, L'_1)}$  is synchronously simulation-based controllable w.r.t.  $G$  and  $\Sigma_{uc}$ . Since  $L_1 \in C_2$  also implies  $L_1 \subseteq L(R)$  and  $L'_1 \subseteq L_m(R)$  and  $R$  and  $G_{(L_1, L'_1)}$  are deterministic, we have  $G_{(L_1, L'_1)} < R$ . Therefore,  $G_{(L_1, L'_1)} \in C_1$ . Next we show that  $R_1 < G_{(L_1, L'_1)}$  for any  $R_1 \in C_1$ . Suppose there is  $R_1 \in C_1$  such that  $R_1 \not< G_{(L_1, L'_1)}$ . Since  $R_1 \in C_1$ , it implies  $R_1 < R$ , moreover,  $R_1$  and  $R$  are deterministic. It follows that  $L(R_1) \subseteq L(R)$  and  $L_m(R_1) \subseteq L_m(R)$ . In addition,  $R_1 \in C_1$  also implies synchronous simulation-based controllability of  $R_1$ . Hence  $L(R_1)$  is language controllable with respect to  $L(G)$  and  $\Sigma_{uc}$  and there is a synchronous simulation relation  $\phi$  such that  $R_1 <_{syn\phi} G$  implying  $L(R_1) \subseteq L(F_{syn}(G))$  and  $L_m(R_1) \subseteq L_m(F_{syn}(G))$  according to Proposition 1. Hence  $L(R_1) \in C_2$ . Moreover,  $L_m(R_1) \subseteq L(R_1)$ . By the definition of supremal, we have  $L(R_1) \subseteq supC_2 = L_1$  and  $L_m(R_1) \subseteq supC_3 = L'_1$ , further,  $R_1$  and  $G_{(L_1, L'_1)}$  are deterministic. It follows that  $R_1 < G_{(L_1, L'_1)}$ , which introduces a contradiction. Hence, the assumption is not correct. That is, we have  $R_1 < G_{(L_1, L'_1)}$  for any  $R_1 \in C_1$ . So  $G_{(L_1, L'_1)} = G_{(supC_2, supC_3)} \in supC_1$ .

Next we present a recursive algorithm for computing the supremal synchronously simulation-based controllable sub-specification.

**Algorithm 2.** *Given a plant  $G$  and a deterministic specification  $R$ , the algorithm for computing the supremal synchronously simulation-based controllable sub-specification with respect to  $G$  and  $\Sigma_{uc}$  is described as follows:*

*Step 1: Obtain  $\det(G) = (X_{\det}, \Sigma, x_{0\det}, \alpha_{\det}, X_{m\det})$ ,  $G' = (F_{syn}(G) \parallel R)_{uc} = (X', \Sigma, x'_0, \alpha', X'_m)$  and  $G'' = G' \parallel \det(G) = (X'', \Sigma, x''_0, \alpha'', X''_m)$ ;*

*Step 2:  $Z_0 := \{(x'_1, x_2) \in X' \times X_{\det} \mid x'_1 = D_d\}$ ;*

*Step 3:  $\forall k \geq 0, Z_{k+1} = Z_k \cup \{z \in X'' - Z_k \mid (\exists \sigma \in \Sigma_{uc}) \alpha''(z, \sigma) \in Z_k\}$ ;*

*Step 4: If  $Z_{k+1} = Z_k \neq Z$ , then the subautomaton  $F_{G''}(X'' - Z_k)$  of  $G''$  is a supremal synchronously simulation-based controllable sub-specification with respect to  $G$  and  $\Sigma_{uc}$ .*

**Theorem 4.** *Algorithm 2 is correct.*

PROOF. Consider  $R'' = F_{G''}(X'' - Z_k) = (Q'', \Sigma, q''_0, \delta'', Q''_m)$ , where  $Z_{k+1} = Z_k \neq Z$  with  $k \geq 0$ . First we show that  $L(R'') \in C_2$ . Definition of  $Z_k$  implies  $L(R'')$  is language controllable w.r.t.  $L(G)$  and  $\Sigma_{uc}$ , and the fact that  $L(\det(G)) = L(G)$  implies  $L(R'') \subseteq L(F_{syn}(G)) \cap L(R)$  and  $L_m(R'') \subseteq L_m(F_{syn}(G)) \cap L_m(R)$ . It follows that  $L(R'') \in C_2$ . Next we show that  $L_2 \subseteq L(R'')$  for any  $L_2 \in C_2$ . Suppose there is  $L_2 \in C_2$  such that  $L_2 \not\subseteq L(R'')$ , that is, there is  $s \in \Sigma^*$  such that  $s \in L_2 \setminus L(R'')$ . Since  $s \notin L(R'')$ , there exists  $s_1 \in \{s\}$  such that  $(x'_1, x_1) \in Z_{k'}$ , where  $x'_1 \in \alpha'(x'_0, s_1)$ ,  $x_1 \in \alpha_{\det}(x_{0\det}, s_1)$  and  $k' = 0, 1, \dots, k$ . Hence there is  $s_2 \in \Sigma_{uc}^*$  such that  $x'_2 \in \alpha'(x'_1, s_2)$  and  $x_2 \in \alpha_{\det}(x_1, s_2)$  with  $(x'_2, x_2) \in Z_0$ , which implies  $s_1 s_2 \in L(G) \setminus L(F_{syn}(G) \parallel R)$ . Moreover,  $L(F_{syn}(G) \parallel R) = L(F_{syn}(G)) \cap L(R)$  and  $L_2 \subseteq L(F_{syn}(G)) \cap L(R)$ . It follows that  $s_1 s_2 \notin L_2$ . If  $s_2 = \epsilon$ , then  $s_1 \notin L_2$ , which implies  $s \notin L_2$ . If  $s_2 \neq \epsilon$ , then  $s_1 s_2(1) \dots s_2(|s_2| - 1) \notin L_2$  because  $L_2$  is language controllable w.r.t.  $L(G)$  and  $\Sigma_{uc}$ ,  $s_2(|s_2|) \in \Sigma_{uc}$  and  $s_1 s_2 \in L(G) \setminus L_2$ . It in turn follows that  $s_1 s_2(1) \dots s_2(|s_2| - 2) \notin L_2$ ,  $s_1 s_2(1) \dots s_2(|s_2| - 3) \notin L_2$ ,  $\dots$ ,  $s_1 \notin L_2$ . Hence  $s \notin L_2$ . So there is a contradiction, which implies the assumption is not correct. Then  $L_2 \subseteq L(R'')$  for any  $L_2 \in C_2$ . As a result,  $L(R'') = \sup C_2$ . It remains to show that  $L_m(R'') = \sup C_3$ . By the definition of  $R''$  and the fact that  $L_m(F_{syn}(G)) \subseteq L_m(G)$ , we have  $L_m(R'') = L(R'') \cap L_m(F_{syn}(G)) \cap L_m(R) = \sup C_2 \cap L_m(F_{syn}(G)) \cap L_m(R) = \sup C_3$ . It follows that  $R''$  is a deterministic automaton such that  $L(R'') = \sup C_2$  and  $L_m(R'') = \sup C_3$ . By Theorem 3, we have  $R'' \in \sup C_1$ .

**Remark 3.** *Algorithm 2 can be terminated because the state set  $X''$  is finite. Because the state numbers of  $F_{syn}(G)$  and  $\det(G)$  are both  $O(2^{|X|})$ . Therefore, the complexity of Algorithm 2 is  $O(2^{2|X|}|Q||\Sigma|)$ .*

Furthermore, the supremal synchronously simulation-based controllable sub-specification can be calculated by formulas without applying the recursive algorithm.

**Theorem 5.** *Given a plant  $G$  and a deterministic specification  $R$ , if  $M = L(R) \cap L(F_{syn}(G)) - [(L(G) - L(R) \cap L(F_{syn}(G)))/\Sigma_{uc}^*]\Sigma^* \neq \emptyset$ , then  $G_{(M,M')}$  is a supremal synchronously simulation-based controllable sub-specification with respect to  $G$  and  $\Sigma_{uc}$ , where  $M' = M \cap L_m(R) \cap L_m(F_{syn}(G))$ .*

PROOF. According to Theorem 1 and Theorem 2 in (Brandt et al., 1990), we obtain  $supC_2 = L(R) \cap L(F_{syn}(G)) - [(L(G) - L(R) \cap L(F_{syn}(G)))/\Sigma_{uc}^*]\Sigma^* = M$ . It follows that  $M' = supC_3$ . From Theorem 3,  $G_{(M,M')}$  is a supremal synchronously simulation-based controllable sub-specification w.r.t.  $G$  and  $\Sigma_{uc}$ .

Now we revisit Example 2.

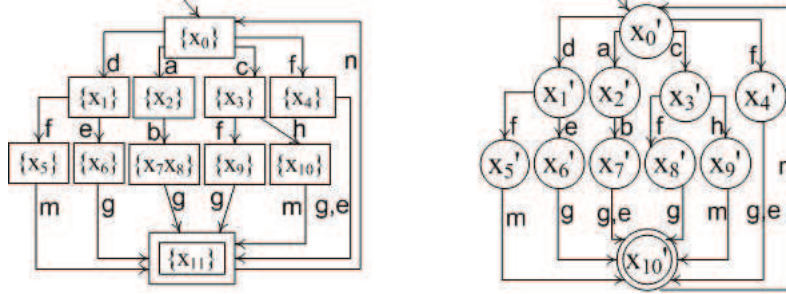


Figure 6:  $F_{syn}(G)$  (Left) and  $det(G)$  (Right)

**Example 3.** *Example 2 indicates that  $R$  is not synchronously simulation-based controllable with respect to  $G$  and  $\Sigma_{uc}$ . Thus, we would like to calculate the supremal synchronously simulation-based controllable sub-specification with respect to  $G$  and  $\Sigma_{uc}$  by the proposed methods.*

(1) *Recursive Method:* From Algorithm 2, we establish  $F_{syn}(G)$  and  $det(G)$ , shown in Fig. 6. Then  $G'' = (X'', \Sigma, x_0'', \alpha'', X_m'') = (F_{syn}(G) || R)_{uc} || det(G)$  is achieved in (Fig. 7 (Left)). We obtain  $Z_0 = \{(D_d, x_{10}')\}$ ,  $Z_1 = Z_0 \cup \{(\{x_7, x_8\}, q_7, x_7'), (\{x_4\}, q_4, x_4')\}$  and  $Z_2 = Z_1 \cup \{(\{x_2\}, q_2, x_2')\} = Z_3$ . Therefore, the supremal synchronously simulation-based controllable sub-specification  $F_{G''}(X'' - Z_2)$  is obtained in Fig. 7.

(2) *Formula-based Method:* First we construct  $F_{syn}(G)$ , which can be seen in Fig. 6 (Left). Hence  $L(R) \cap L(F_{syn}(G)) = \overline{(d(fm + eg)n + c f g n + f g n)^* ab}$ . Thus,

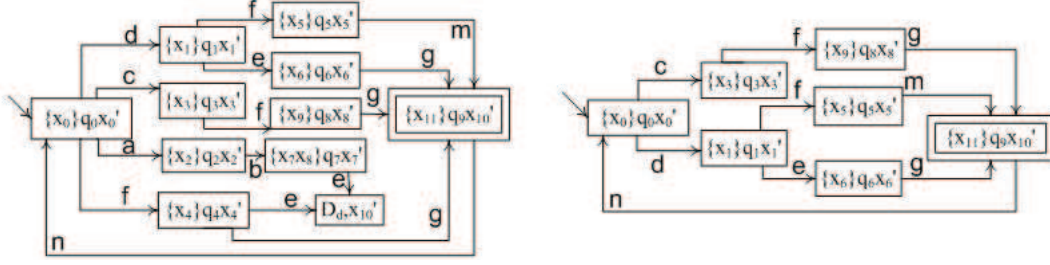


Figure 7:  $(F_{syn}(G)||R)_{uc}|det(G)$  (Left) and  $F_{G''}(X'' - Z_2)$  (Right)

$M = L(R) \cap L(F_{syn}(G)) - [(L(G) - L(R) \cap L(F_{syn}(G))) / \Sigma_{uc}^*] \Sigma^* = \overline{(d(fm + eg)n + c fgn + fgn)^* ab - (d(fm + eg)n + c fgn + fgn)^* ab \Sigma^* - (d(fm + eg)n + c fgn + fgn)^* a \Sigma^* - (d(fm + eg)n + c fgn + fgn)^* f \Sigma^* = (d(fm + eg)n + c fgn)^* \neq \emptyset}$  and  $M' = M \cap L_m(R) \cap L_m(F_{syn}(G)) = (d(fm + eg)n + c fgn)^* (d(fm + eg) + c f g)$ . The supremal synchronously simulation-based controllable sub-specification  $G_{(M, M')} = F_{G''}(X'' - Z_2)$  is achieved in Fig. 7 (Right).

## 6. Conclusion

In this paper, we investigated the bisimilarity enforcing supervisory control of nondeterministic plants for deterministic specifications. A necessary and sufficient condition for the existence of a bisimilarity enforcing supervisor was deduced from synchronous simulation-based controllability of the specification, which can be verified by a polynomial algorithm. For those specifications fulfilling the existence condition, a bisimilarity enforcing supervisor has been constructed. Contrarily, when the existence condition does not hold, a recursive method and a formula-based method have been developed to calculate the maximal permissive sub-specifications.

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